Chapter 11 – Sine and Cosine (S and ST scale)

On the S scale the graduations in black are for sines. Most Slide Rules have graduations in red for Cosines, these reading from right to left.

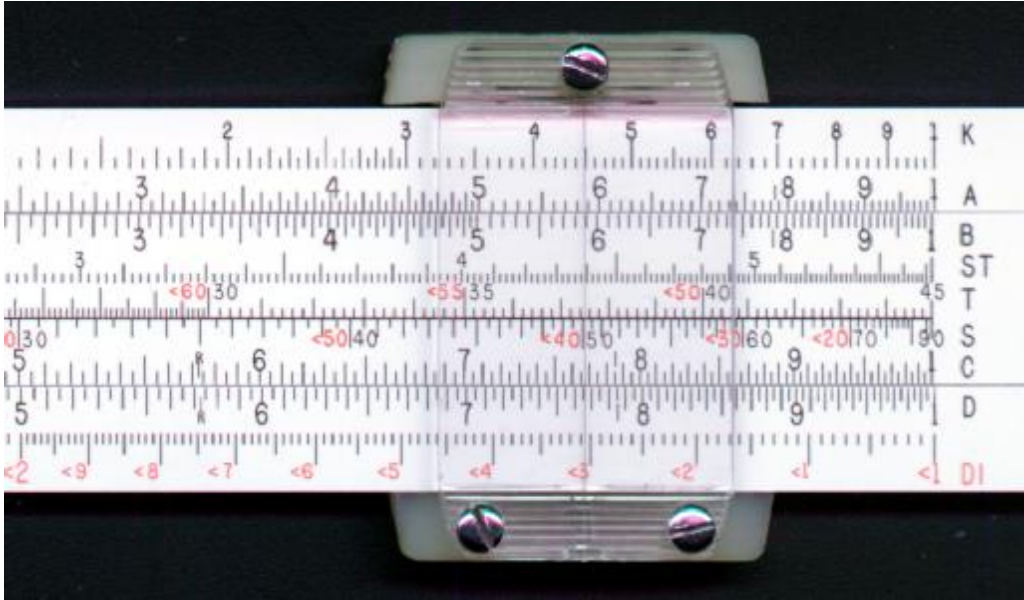**11.1 Sine (S scale – for angles between 5°44' and 90°**

Fig 11-1

Example: sin 50° = 0.766 (Fig. 11-1)
1. Set the hair line over 50° (in black) on the S scale.
2. Under the hair line read off 0.766 on the D scale as the answer.

Note:
   (a) The sines of angles between 5°44' and 90° are read directly off the D scale as values between 0.1 and 1.
   (b) The subdivisions on the S scale (i.e. graduations between the degree marks) on most rules allow us to express a fraction of a degree as either a decimal of a degree or so many minutes. That is, there are 10, 5, or 2 subdivisions.
   (c) There are two limitations in using the S scale:
       a. We cannot obtain the sines of angles less than 5°44'. (See 11.2 for the use of the ST scales for the sines of small angles.)
       b. The lack of accuracy for angles close to 90° (see Unit 13 for better method involving P scale for sines of angles close to 90°.)
   (d) For angles greater than 90°, convert them to an equivalent expression involving and angle less than 90°, as is the usual practice.
       e.g.   sin 150° = sin (180°-30) = sin 30°
               sin 240° = sin(180°+60) = - sin 60°
               sin 315° = sin(360°-45°) = - sin 45°
   (e) If your Slide Rule has on S' scale on the slide, this is the same scale as the S, and is best used in conjunction with the C' scale (which is also on the slide).
   (f) If the value of the sine of an angle is known, then the angle can be obtained by:
       a. Setting the hair line over the 'value' on the D scale.
       b. Under the hair line read off the angle on the S scale.
       (This of course is evaluating inverse trigonometrical functions, e.g. $\sin^{-1} 0.866 = 60°$)

Exercise 11(a)

(i)	sin 21° =	(v)	sin 120° =
(ii)	sin 36° 36' =	(vi)	sin 300° =
(iii)	sin 56° 30' =	(vii)	sin x = 0.274
(iv)	sin 7° 21' =	(viii)	sin x = 0.855

11.2 Sine (ST scale – for angles less then 5°44'

For sines of angles between 0.573° (or 34') and 5.74° (or 5°44') we find the angle on the ST scale and read the sine of the angle off the D scale as a value between 0.01 and 0.1. For the sines of angles between 0.0573° and 0.573° we find the angles on the ST scale, reading the graduations not as 0.9, 1.5, 5.5, etc. but as 0.09, 0.15, 0.55 etc. (i.e. $\frac{1}{10}$ of their marked value). The sine of such angles is read off the D scale as a value between 0.001 and 0.01. This can be repeated for even smaller angles if desired. For more detailed explanation and further uses of the ST scale for very small angles, see Unit 14. In using the ST scales note that minutes have to be expressed as a decimal fration of a degree (i.e. by dividing by 60).

Example: sin 0.82° = 0.0143 (Fig. 11-2)
1. Set the hair line over 0.82° on the ST scale.
2. Under the hair line read off 0.0143 on the D scale as the answer.

Note: If we required sin 0.082°, we would locate the hair line as above, and the answer would be then read off as 0.00143.

Exercise 11(b)

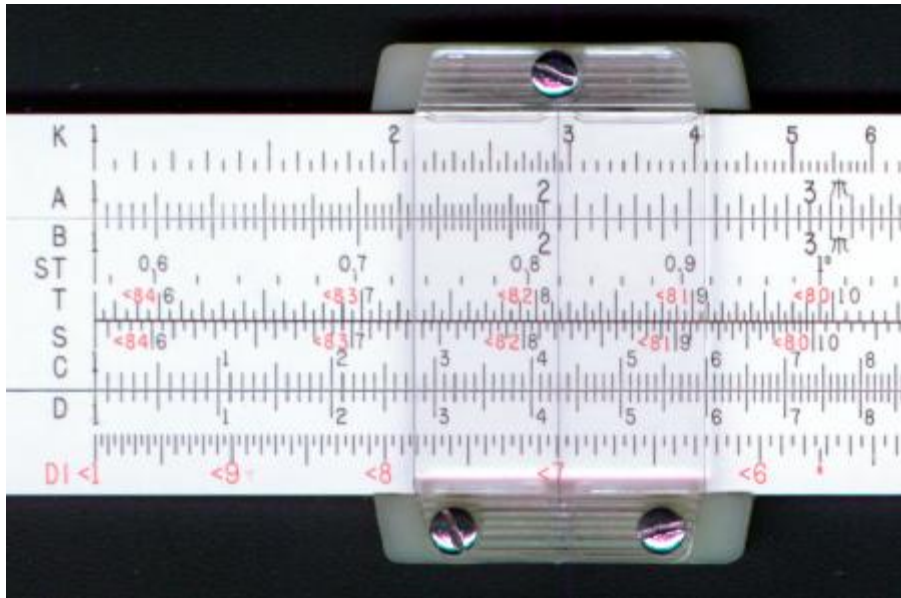|				
(i)	sin 4°31' =	(iv)	sin 30' =
(ii)	sin 2.3° =	(v)	sin 0.2° =
(iii)	sin 0.79° =	(vi)	sin 9' =

Fig 11-2

11.3 Cosine (S and ST scale)

As cos θ = sin (90 - θ) (e.g. cos 60 = sin 30, etc.) we find the cosine of an angle by looking up the sine of its complement. To facilitate this, the angles for cosines are usually marked on the S scale in red.
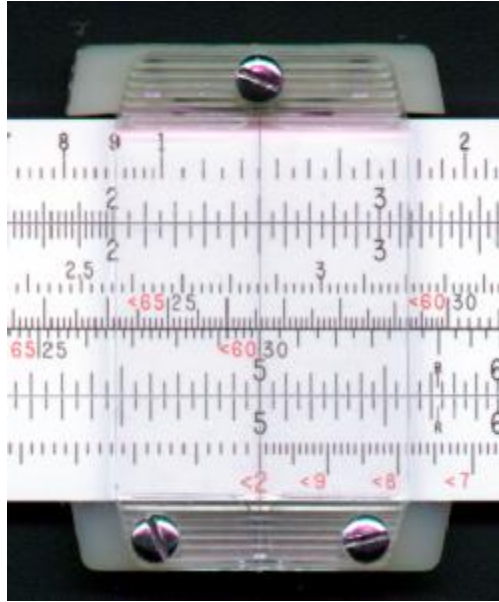
Fig 11-3

Example 1: Cos 42° = 0.743 (Fig 11-3)
  1.   Set the hair line over the 42° (for red graduations) on the S scale.
  2.   Under the hair line read off 0.743 on the D scale as the answer.

Note: The S scale allows us to obtain cosines of angles between 0° and 84°16'. For angles greater than 84°16' we can use the ST scales as follows: --

Example 2: cos 88° = 0.0349
  1.   Set the hair line over 2° on the ST scale. (as cos 88° = sin 2°).
  2.   Under the hair line read off 0.0349 on the D scale as the answer.

Exercise 11(c)

(i)	cos 30° =		(v)	cos 89° =
(ii)	cos 79° =		(vi)	cos 89°30' =
(iii)	cos 65.4° =		(vii)	cos x = 0.4
(iv)	cos 85°30' =		(viii)	cos x = 0.04

11.4 Cosecant and Secant
We recall that:

$$\text{cosec } \theta = \frac{1}{\sin \theta} \quad \text{and sec } \theta = \frac{1}{\cos\theta}$$

Thus, to find cosecant or secant we look them up the same as for sine and cosine (using S or ST scales) We read the answer off on the reciprocal scales DI or CI instead of the D or C scale.

Example 1: cosec 15° = 3.86
 (i)   Set the hair line over 15° on the S scale.
 (ii)   Under the hair line read off 3.86 on the DI (or CI) scale as the answer.

Example 2: sec 59° = 1.94
  1.   Set the hair line over 59° (for red graduations) on the S scale.
  2.   Under the hair line read off 1.94 on the DI (or CI) scale as the answer.

Note: As the value of sine and cosine is always less than 1, the value of their reciprocals cosecant and secant will always be greater than 1.

Exercise 11(d)

(i)	cosec 45° =	(iv)	sec 82° =
(ii)	cosec 24°12' =	(v)	sec 22° =
(iii)	cosec 3.5° =	(vi)	sec 2°30' =

11.5 Multiplication and Division with Sines and Cosines

The following table gives a few possible calculations involving sines and cosines. It will be observed that the S' scale on the slide is extremely handy to use in combination with the usual S scale on the body of the Slide Rule. If the answer runs off the end D scale make use of the DF scale for the answer. Also note for small angles the ST scale would be used where the table indicates the S scale. In the following 'H.L.' stands for hair line.)

Example	Set HL Over	Under HL Place	Reset HL over	Under HL answer
$a \sin \theta$	θ on S scale	Index of C scale	a on C scale	on D scale
$a \cos \theta$	θ(red) S	Index C	a C	D
$(a \sin \theta)^2$	θ S	Index C	a C	A
$(a \cos \theta)^2$	θ(red) S	Index C	a C	A
$\dfrac{a}{\sin \theta}$	θ S a C	a C θ S'	Index D Index C	C D
$\dfrac{a}{\cos \theta}$	θ(red) S	a C	Index D	C
$\dfrac{\sin \theta}{a}$	θ S	a C	Index C	D
$\dfrac{a \sin \theta}{b}$	θ S	b C	a C	D
$ab \cos \theta$	θ(red) S	a CI	b C	D
$\dfrac{1}{\sin^2 \theta}$	θ S θ S	index C	Index A	B BI
$\sin \theta \sin \phi$	θ S	Index C	ϕ S'	D
$\left(\cos \theta \cos \phi\right)^2$	θ(red) S	Index C	ϕ(red) S'	A
$\dfrac{\sin \theta}{\sin \phi}$	θ S	ϕ S'	Index C	D
$\dfrac{\cos^2 \theta}{\cos^2 \phi}$	θ(red) S	ϕ(red) S'	Index C	A
$\pi \sin \theta$	θ S			DF
$\dfrac{\sin \theta}{\pi}$	θ S	π C	Index C	D
$\pi \cos^2 \theta$	θ(red) S	Index B	π C	A
$\sqrt{\pi} \sin \theta$	θ S	Index C	π B	D
$\dfrac{\sin \theta \sin \phi}{\sin \alpha}$	θ S	α S'	ϕ S'	D

Exercise 11(e)

(i) $3.4 \sin 27° =$

(ii) $2.7 \cos 60° =$

(iii) $(3.4 \sin 27°)^2 =$

(iv) $\dfrac{1.4}{\sin 56°} =$

(v) $\dfrac{6.4}{\cos 74°30'} =$

(vi) $\dfrac{5.46 \sin 69°}{\sin 12°} =$

(vii) $\sin 14° \times \sin 26° =$

(viii) $(\cos 59° \times \cos 75°)^2 =$

(ix) $\left(\dfrac{\sin 70°}{\sin 21°}\right)^2 =$

(x) $\dfrac{1}{\cos^2 36°30'} =$

(xi) $\pi \sin 21°20' =$

(xii) $\dfrac{\cos 76°45'}{\pi}$

(xiii) $\pi \sin^2 81° =$

(xiv) $\sqrt{\pi} \sin 32°12' =$

(xv) $\dfrac{\sin 29° x \sin 34°}{\sin 36°} =$

(xvi) $\dfrac{\cos 67° x \cos 53°}{0.031} =$